Magnetic and rotational deformation of strange quark stars

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February 21, 2023
World Copernican Congress, Toruń

A brief story about strange quark stars

1. **1960s**, in Stanford Linear Accelerator Center **SLAC** a high-energy scattering examination showed that protons and neutrons are composed of smaller particles.
2. **1964, Gell–Mann & Zweig** proposed the theory of quarks to explain these subatomic particles.
3. Bodmer **(1971)**, Terazawa **(1979)**, and Witten **(1984)** pointed out that the strange quark matter may be the stable state of matter (Energy per baryon $\leq$in ^{56}Fe).
4. **1984, Farhi & Jaffe** used the MIT bag model to study the stability of strange quark matter
5. **1986, Alcock, et al.** and **Haensel, et al.** independently, discussed the properties of strange stars.
6. **1995, Weber** discussed quark deconfinement in the core of neutron stars.
7. **Rapidly rotating strange star** is discussed by Gondek-Rosińska, et al. in 2000, **Maximum rotational frequency of NS and SQS** is studied by Haensel et al. in 2009.
8. Deformation of a magnetized neutron star was studied by Mallick, et al. in 2014 and Mastrano, et al. in 2015. A magnetized rotating neutron star is studied by Chatterjee et al. in 2015.

Some theoretical properties of strange stars and neutron stars

Quark Stars	**Neutron Stars**
Made entirely of deconfined up, down, strange quarks, and electrons	Nucleons, hyperons, boson condensates, deconfined quarks, electrons, and muons
Quarks ought to be color superconducting	Superfluid neutrons Superconducting protons
Energy per baryon $\lesssim 930$ MeV	Energy per baryon > 930 MeV
Self-bound ($M \propto R^3$)	Bound by gravity
Maximum mass $\sim 2\, M_\odot$	Same
No minimum mass	$\sim 0.1\, M_\odot$
Radii $R \lesssim 10 - 12$ km	$R \gtrsim 10 - 12$ km

F. Weber et al. (2012)

Based on the paper: F. Kayanikhoo & M. Kapusta, et al. (In preparation)

Strange quark stars model

- We made 32 setups by varying the strength of **magnetic field** from 0 to 10^{18} G in the **rotational frequencies** of 0 Hz, 400, 800, and 1200 Hz.

Equation of state

- The density-dependent MIT bag model
- Strange quark matter (**SQM**) contains **up, down and strange** quarks.
- Landau quantization effect

The total energy density of SQM:

$$\varepsilon_{tot} = \sum_{i,\pm} \varepsilon_i^{\pm} + \mathcal{B}_{bag} \tag{1}$$

- $\varepsilon_i^{\pm}$ is the kinetic energy density, i represent quarks and $\pm$ is the spin of quarks. $\varepsilon_i^{\pm}$ is computed using the **Fermi relations considering the Landau quantization effect**.
- $\mathcal{B}_{bag}$ is the bag constant:

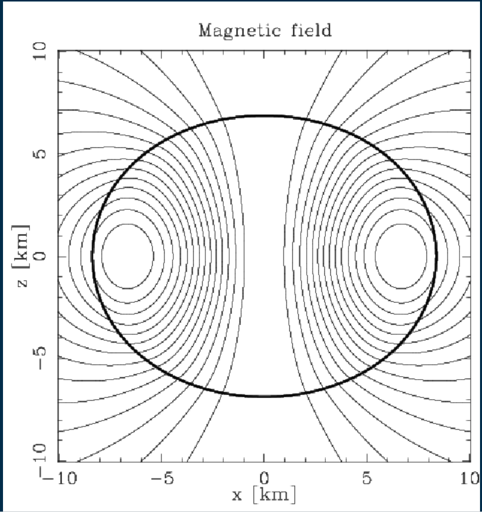
$$\mathcal{B}_{bag}(\rho) = \mathcal{B}_{\infty} + (\mathcal{B}_0 - \mathcal{B}_{\infty}) \exp\left(-\alpha \frac{\rho^2}{\rho_0^2}\right) \tag{2}$$

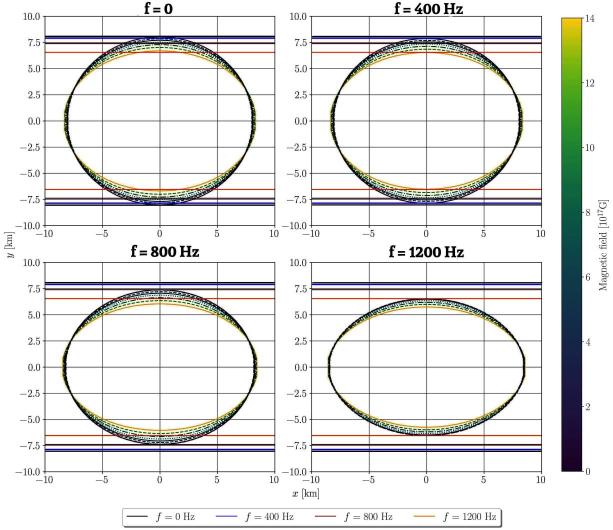
  $\alpha = 0.17$, and $\mathcal{B}_0 = \mathcal{B}_{bag}(0) = 400\ \mathrm{MeV/fm^3}$. We should define $\mathcal{B}_{\infty}$ in a way that bag constant be compatible with experimental data (CERN-SPS). We determine $\mathcal{B}_{\infty} = 8.99$ $\mathrm{MeV/fm^3}$ by putting quark energy density = hadronic energy density.
- **Equation of state:**

$$P(\rho) = \rho \left(\frac{\partial \varepsilon_{tot}}{\partial \rho}\right) - \varepsilon_{tot} \tag{3}$$

Numerical method

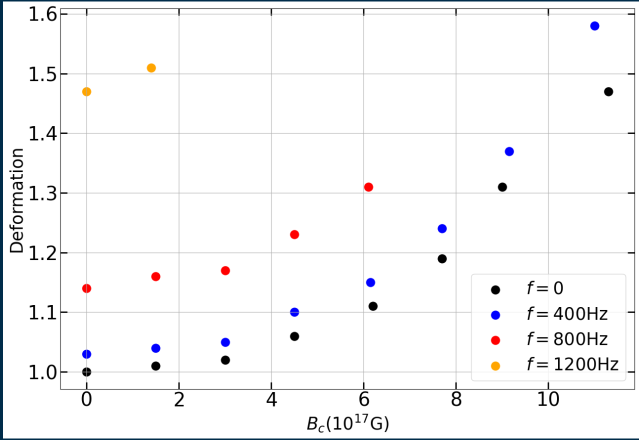
- We solve a set of four elliptic partial differential equations that are derived by solving Einstein's equation in **axisymmetric space-time** using **LORENE library** http://www.lorene.obspm.fr.
- LORENE uses state of **spectral methods** to solve partial differential equations which makes calculations much more accurate than grid-based methods.





- **Color** of the ellipses indicates the **magnitudes of the central magnetic field**.
- **Straight lines**, which at the same positions in all panels, indicate the **polar radius** of **non-magnetized configuration** in each rotational frequency
- The **polar radius** is **decreasing** with increasing magnetic field and rotational frequency.

Magnetic and rotational deformation



- Deformation parameter is $\boldsymbol{a = R_{eq}/R_{pol}}$
- $\boldsymbol{a}$ changes from **1** ($f = 0$ Hz, $B = 0$) to **1.58** ($f = 400$ Hz, $B_c \sim 5 \times 10^{18}$ G)
- We found the deformation parameter of stable configurations is fitted with the following relation:

$$\boldsymbol{a \simeq (1 + bB_c^2)(1 + cf^2)}$$

where $\boldsymbol{B_c}$ is the central magnetic field in the unit of 10^{17} G and $\boldsymbol{f}$ is rotational frequency in Hz. The coefficients are $b = 3.2 \times 10^{-3}$ and $c = 2.34 \times 10^{-7}$.

Summary

- We made 32 setups by varying the strength of **magnetic field** from 0 to 10^{18} G in the **rotational frequencies** of 0 Hz, 400, 800, and 1200 Hz.
- Deformation parameter $a = R_{eq}/R_{pol}$ increases with increasing magnetic field and rotational frequency.
- We found a simple function to be fitted on the deformation parameter,

$$a \simeq (1 + bB_c^2)(1 + cf^2)$$

  - If we spin up the non-magnetized SQS to **1200** Hz, it deforms with the parameter $a \simeq 1.48$.
  - By spinning up the magnetized SQS with $B_c = 10^{18}$ G to **1200** Hz, the deformation parameter changes from **1.32** to **1.76**.