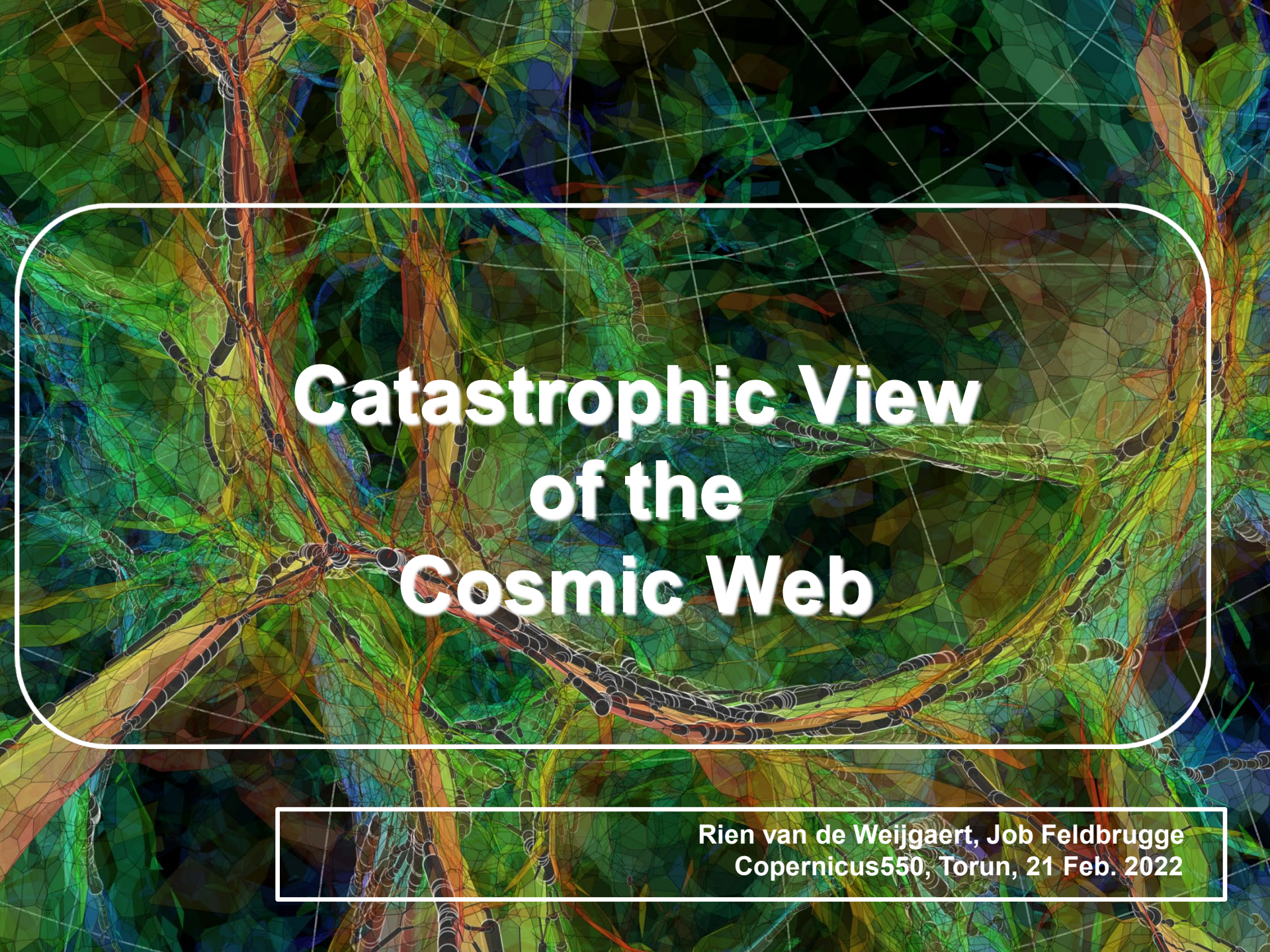


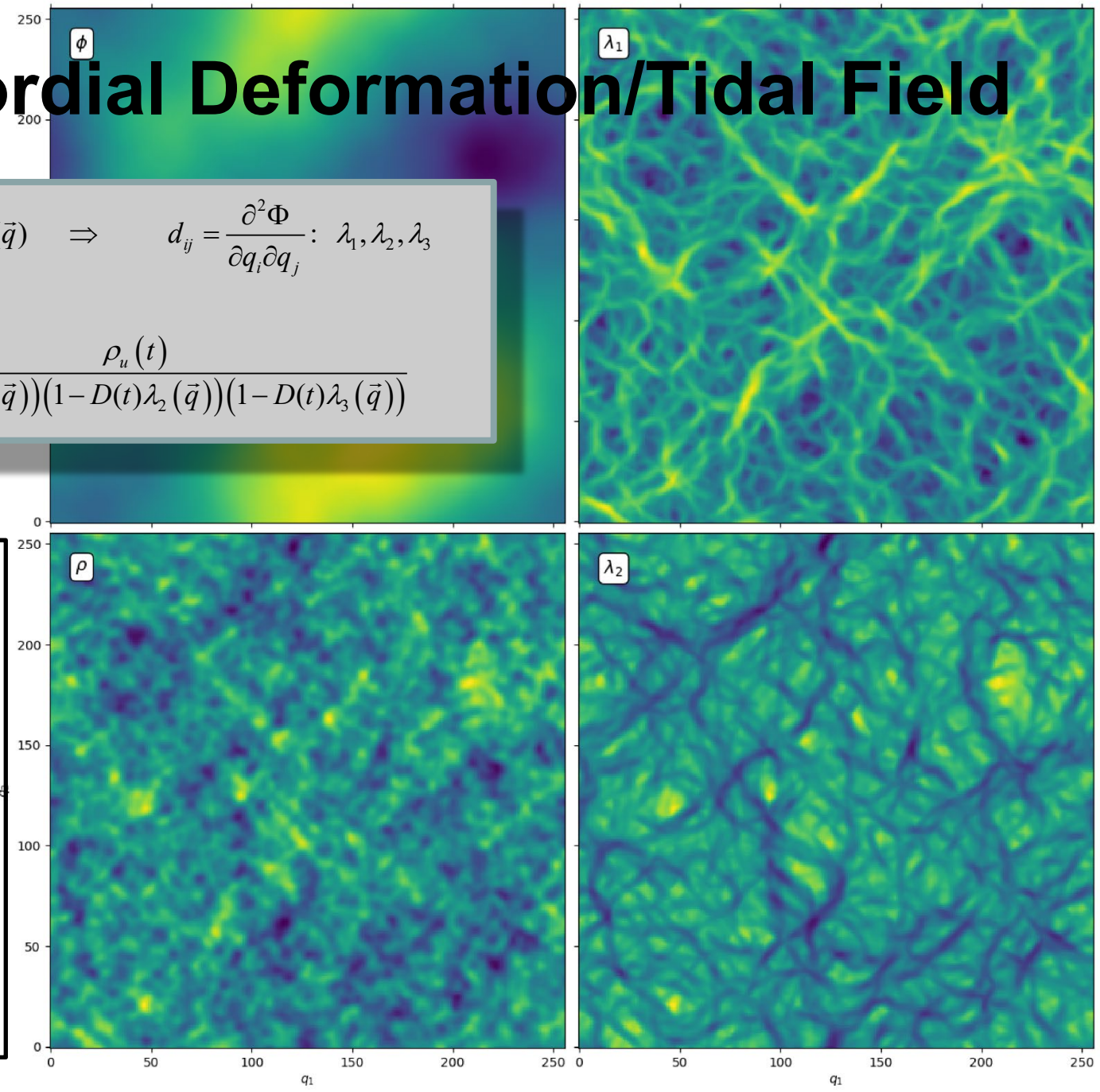
# Catastrophic View of the Cosmic Web

Rien van de Weijgaert, Job Feldbrugge
Copernicus550, Torun, 21 Feb. 2022

# Primordial Deformation/Tidal Field

$$\vec{x}(\vec{q},t) = \vec{q} - D(t)\vec{\nabla}\Phi(\vec{q}) \quad \Rightarrow \quad d_{ij} = \frac{\partial^2 \Phi}{\partial q_i \partial q_j} : \ \lambda_1, \lambda_2, \lambda_3$$

$$\rho(\vec{q},t) = \frac{\rho_u(t)}{\left(1 - D(t)\lambda_1(\vec{q})\right)\left(1 - D(t)\lambda_2(\vec{q})\right)\left(1 - D(t)\lambda_3(\vec{q})\right)}$$



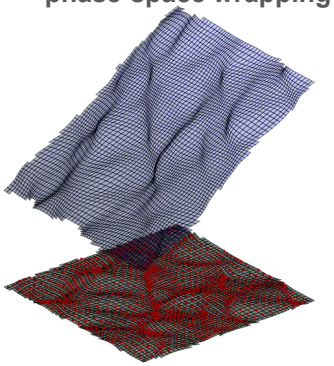

**Primordial Tidal/Deformation Eigenvalue Landscape**
(initial Gaussian field)

**Connectivity Cosmic Web:**

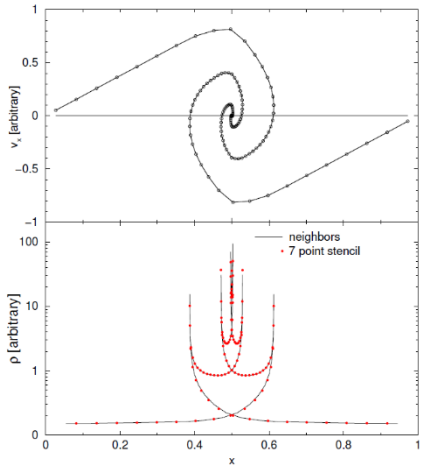
**Topology Tidal/deformation field**

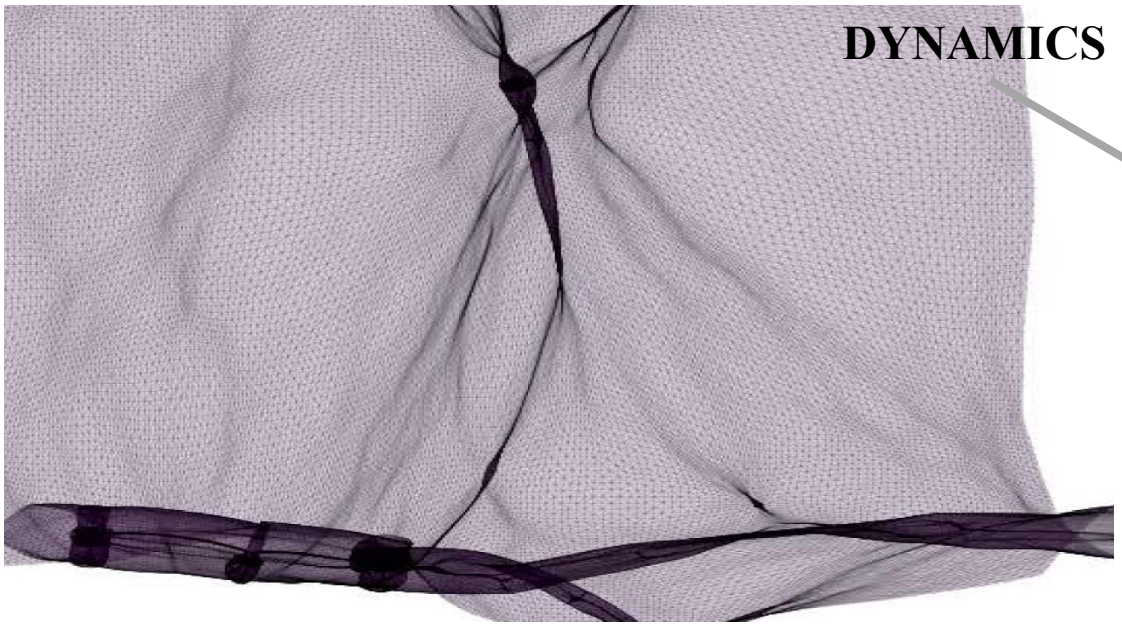
# Caustic Skeleton & Phase-Space Wrapping

Cosmic Web
phase-space wrapping



Hidding 2014



**DYNAMICS**

**Cosmic Catastrophe Theory:**

Lagrangian catastrophe/caustic classification V. Arnold

# Deformation, Streaming & Caustics

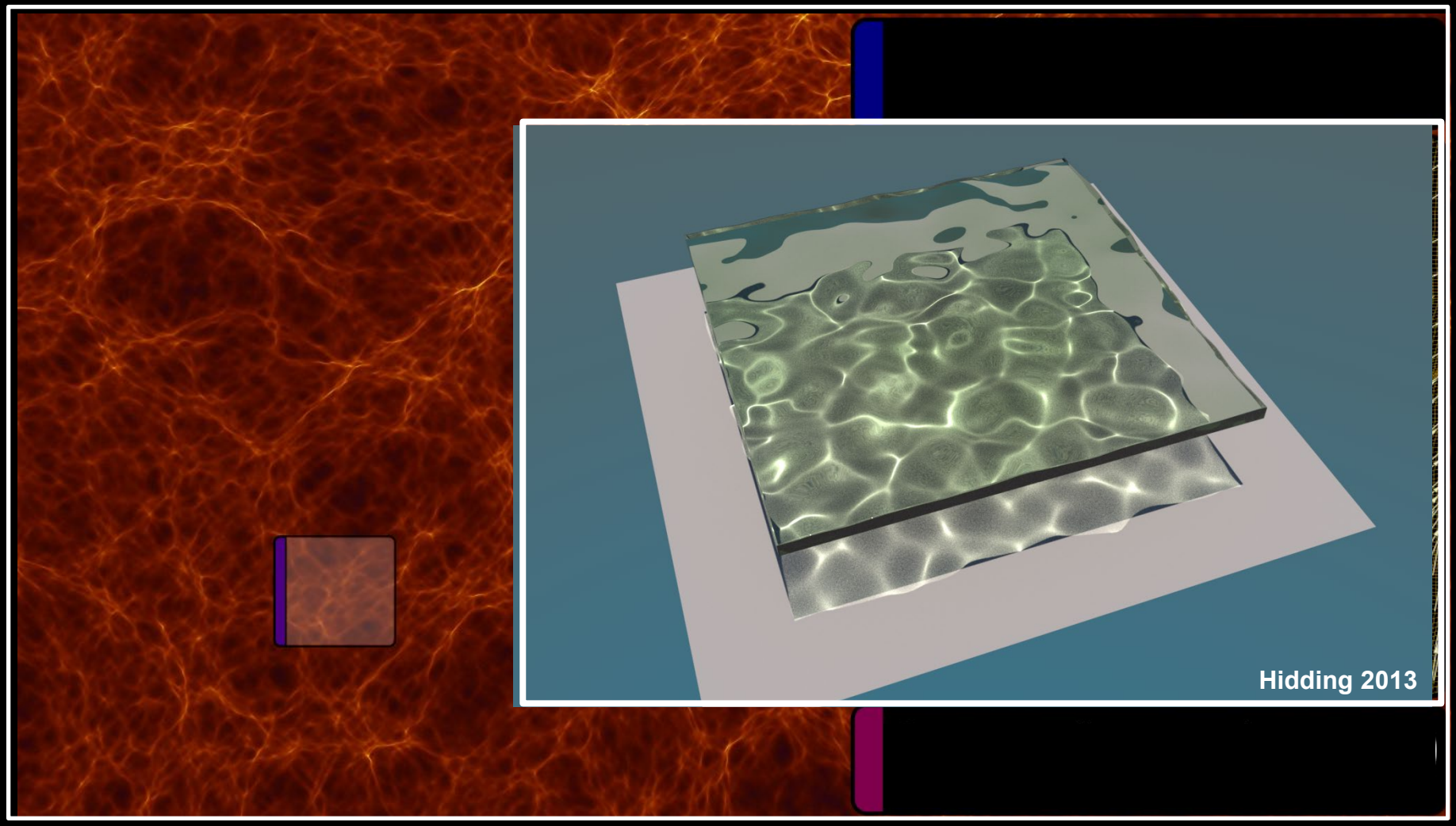
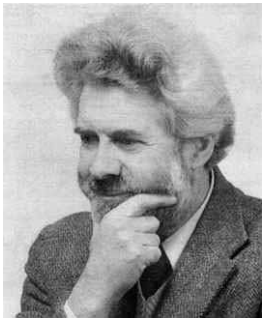


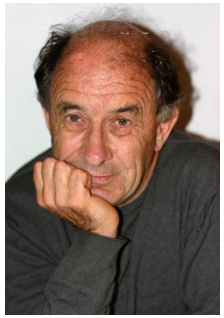

Illustration of the formation of caustics due to streaming paths of light through deforming medium

# Leaders of Catastrophe



E. Zeeman

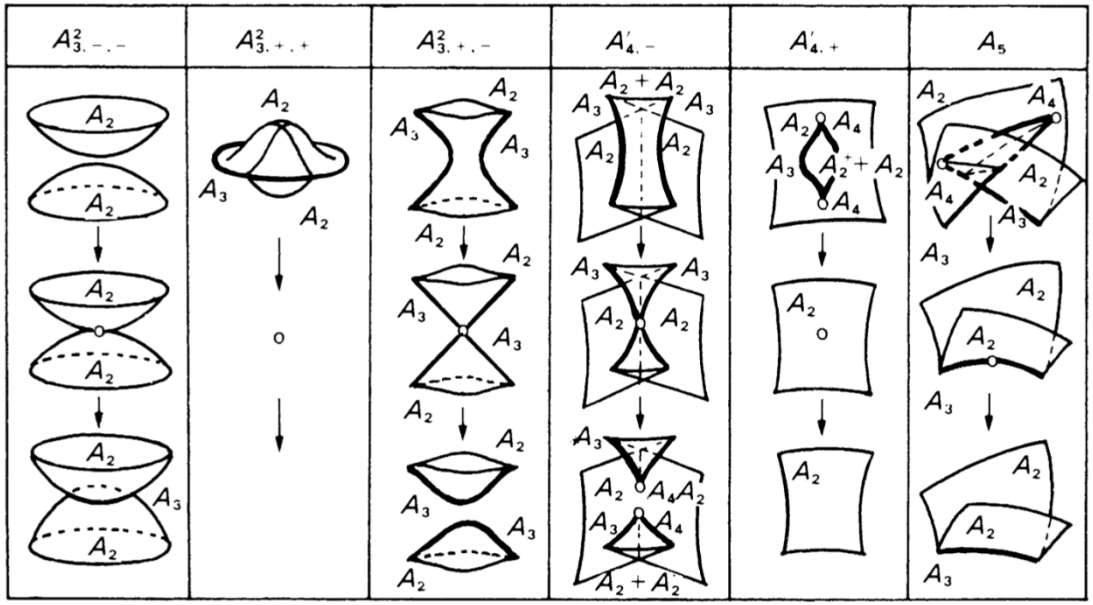
R. Thom



V.I. Arnol'd

**Arnold V.I. (and others):**
Caustic classification on basis of Normal Forms:

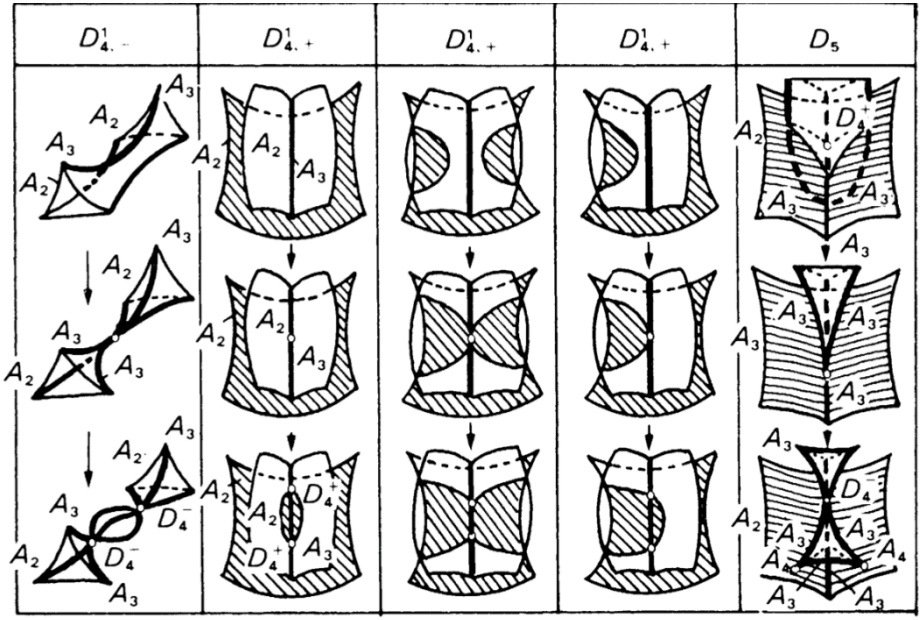
$$F(q_1) = q_1^{\,5} + \lambda q_1^{\,3} + \mu q_1^{\,2}$$




**Arnold V.I., 1986,**
Catastrophe Theory, Springer

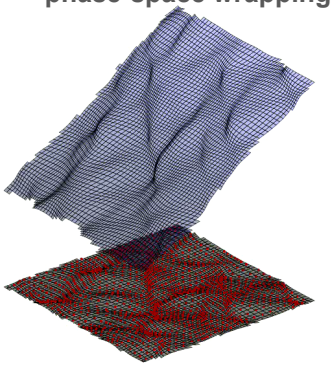
**Arnold V.I., Shandarin S.F., Zeldovich Ya.B., 1982**
The Large Scale Structure of the Universe I. General Properties. One and Two-dimensional models Geophys. Astrophys. Fluid Dynamics, 20, 1-2

**Arnold V.I., 1986,**
Evolution of singularities of potential flows in collisionless-free media and the metamorphosis of caustics in three-dimensional space

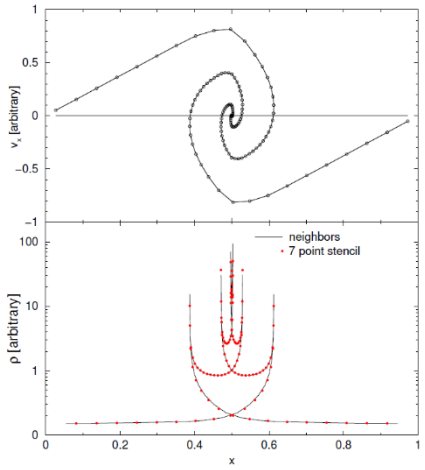


# Caustic Skeleton & Phase-Space Wrapping

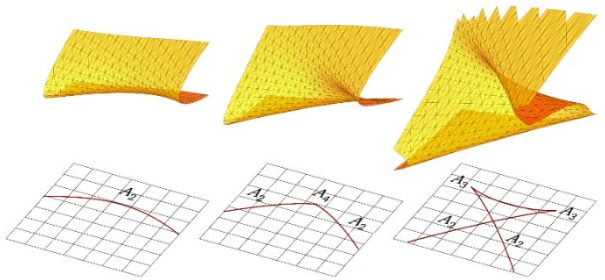
Cosmic Web
phase-space wrapping



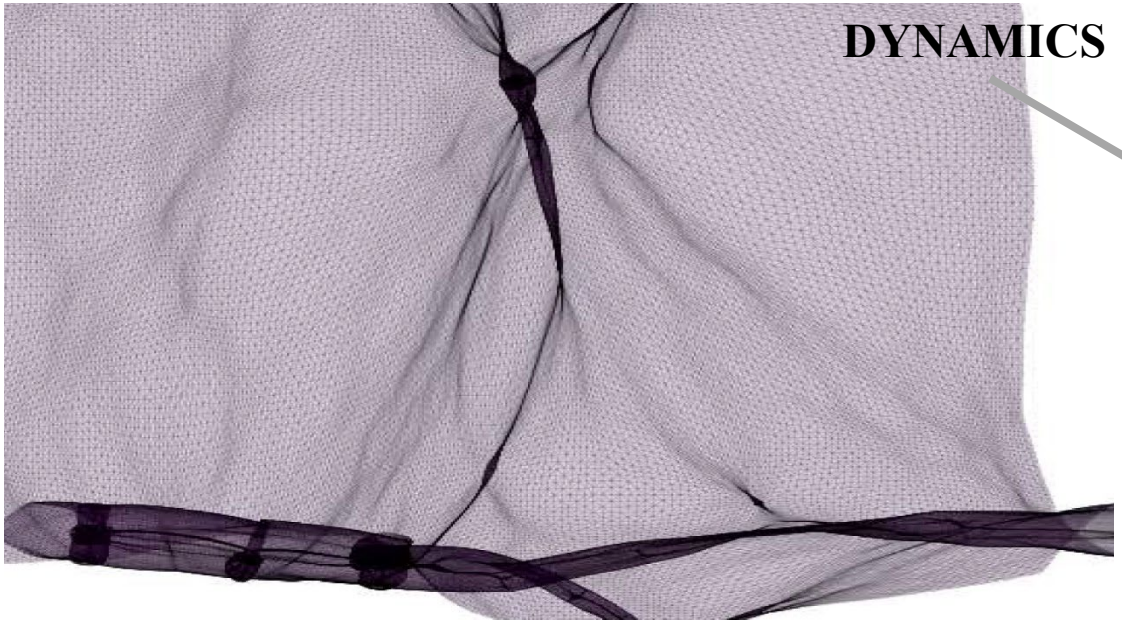
Hidding 2014




Classification phase-space folding –
Structural Morphology



Feldbrugge, vdW et al., JCAP, 2018
Hidding, Shandarin, vdW, MNRAS, 2014
Feldbrugge & vdW, JCAP, 2022/23, in press
Feldbrugge,




## Cosmic Catastrophe Theory:

### Lagrangian catastrophe/caustic classification V. Arnold

In Lagrangian space (coordinates q):

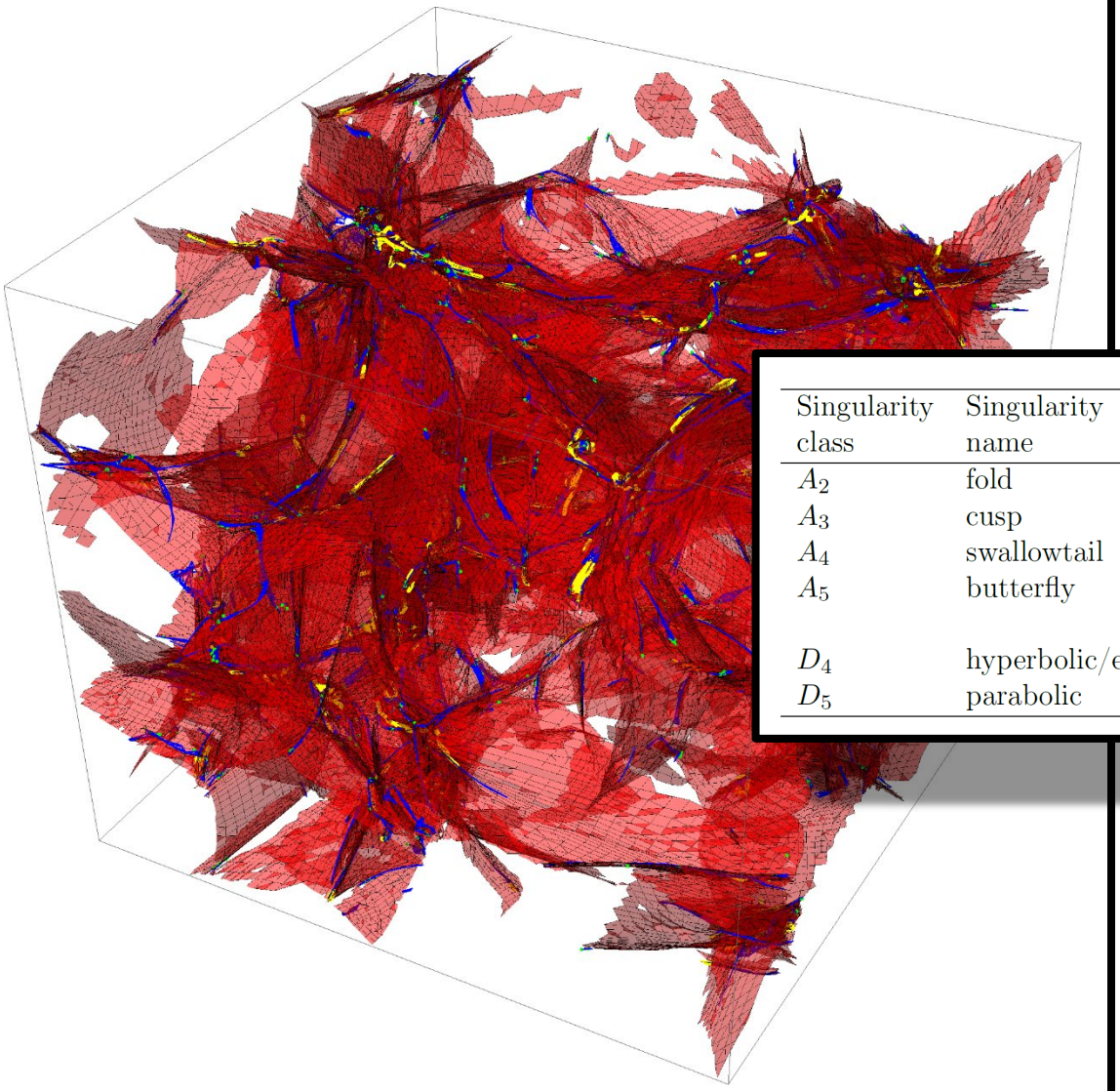
A singularity forms in a manifold M at location $q_s$ when at $q_s$,

- the deformation tensor eigenvalue $\mu_i(q_s)$
- the corresponding eigenvector $\vec{v}_i(q_s)$

when at least one nonzero tangent vector $\vec{T}$

$$\{1+\mu_i(q_s)\}\ \vec{v}_i^{*}(q_s)\cdot\vec{T}=0$$

# Cosmic Web - Caustic Skeleton



**Cosmic Web:** **Caustic features**

**Deformation field** **(Gaussian initial density field)**

| Singularity class | Singularity name | Feature in the 2D cosmic web | Feature in the 3D cosmic web |
|---|---|---|---|
| $A_2$ | fold | collapsed region | collapsed region |
| $A_3$ | cusp | filament | wall or membrane |
| $A_4$ | swallowtail | cluster or knot | filament |
| $A_5$ | butterfly | not stable | cluster or knot |
| $D_4$ | hyperbolic/elliptic | cluster or knot | filament |
| $D_5$ | parabolic | not stable | cluster or knot |

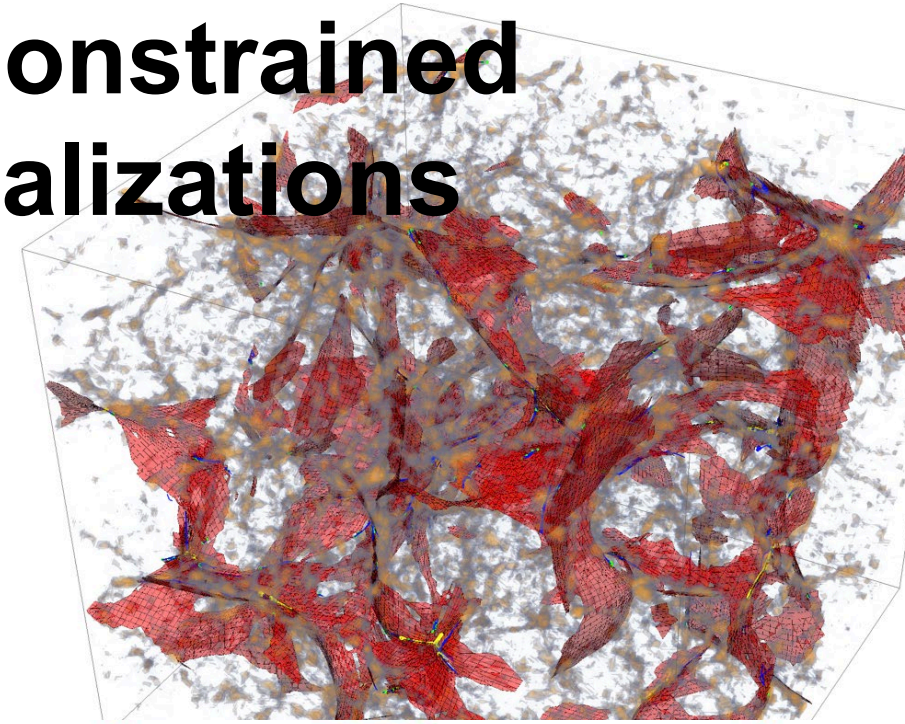
**Caustic connectivity:**

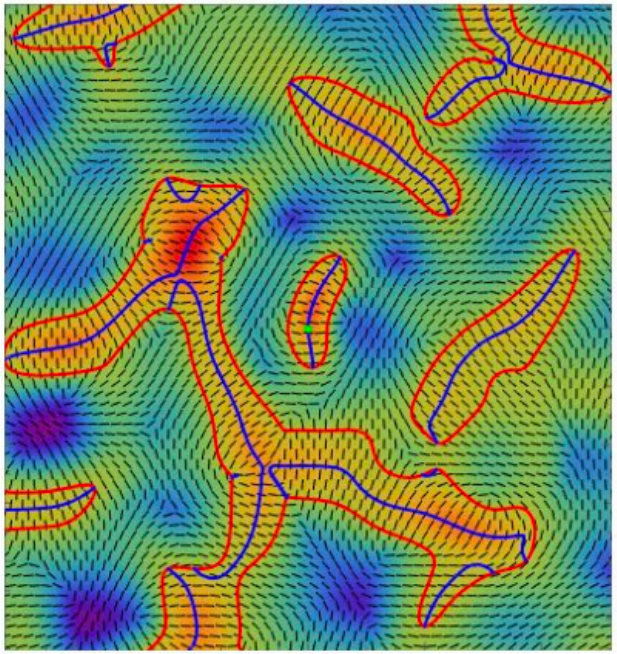
Topology of the deformation field

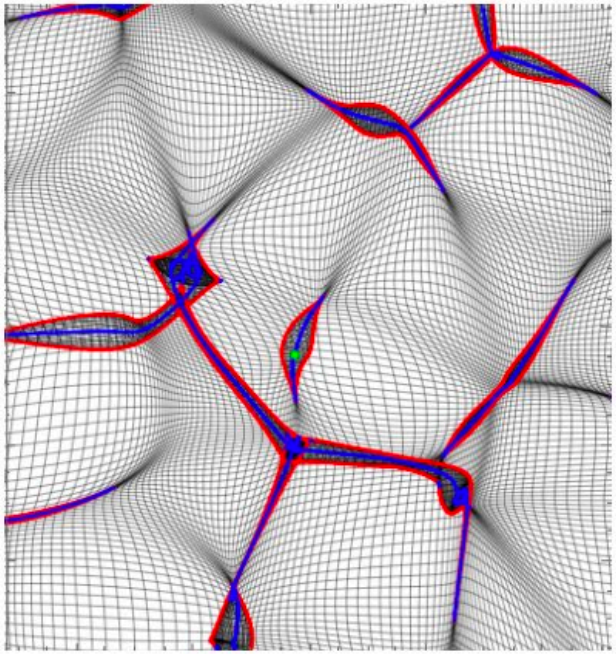
Feldbrugge & vdW 2022/2023, JCAP, in press

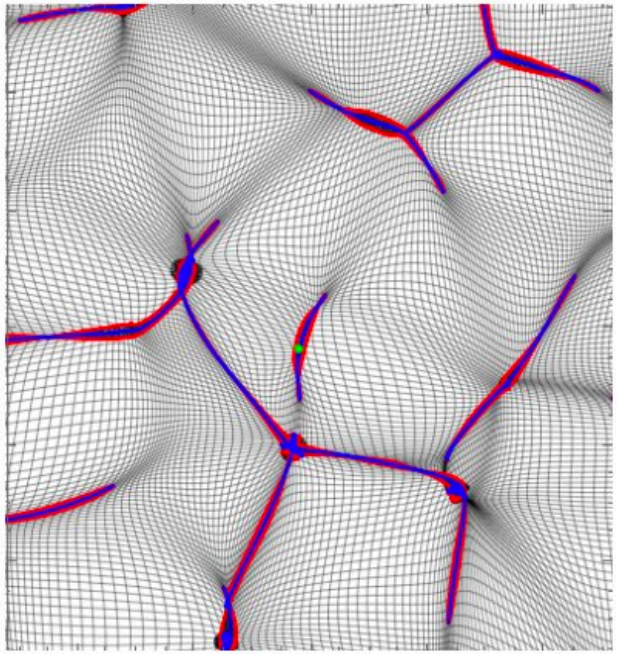
# (Nonlinear) Constrained Filament Realizations

- Assessing the mass distribution in and around the caustic spine of the cosmic web:
  - What are the properties of walls/filaments/clusters?
  - What is their mass distribution?
  - How do they form and relate to the initial conditions?
  - Detailed merging history and hierarchical evolution of the filamentary network?



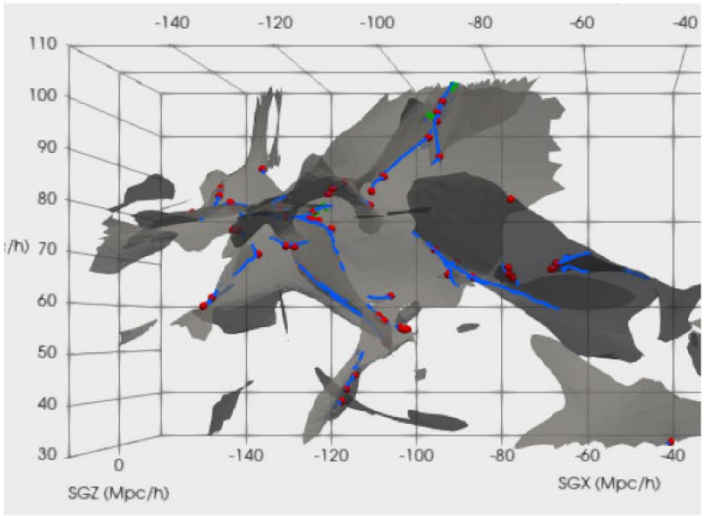






# Catastrophic View of the Local Universe

**Bixerman, Feldbrugge, vdW et al 2023**



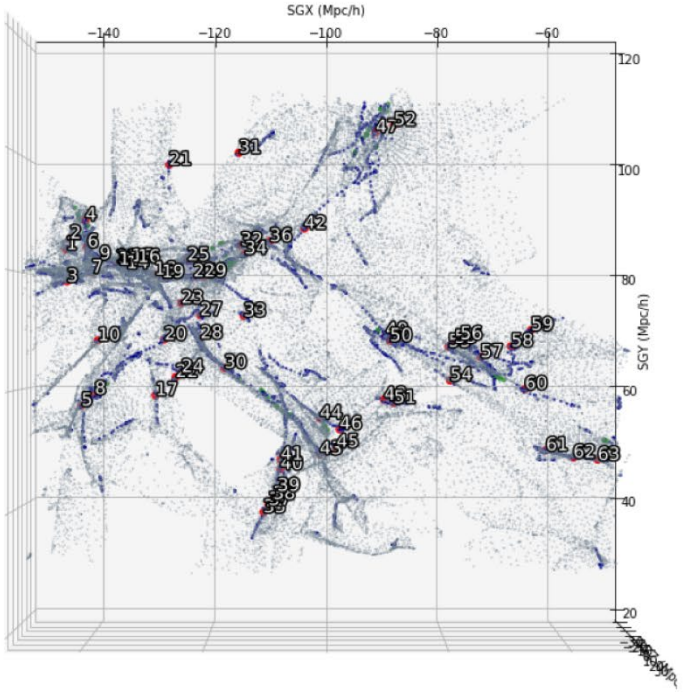
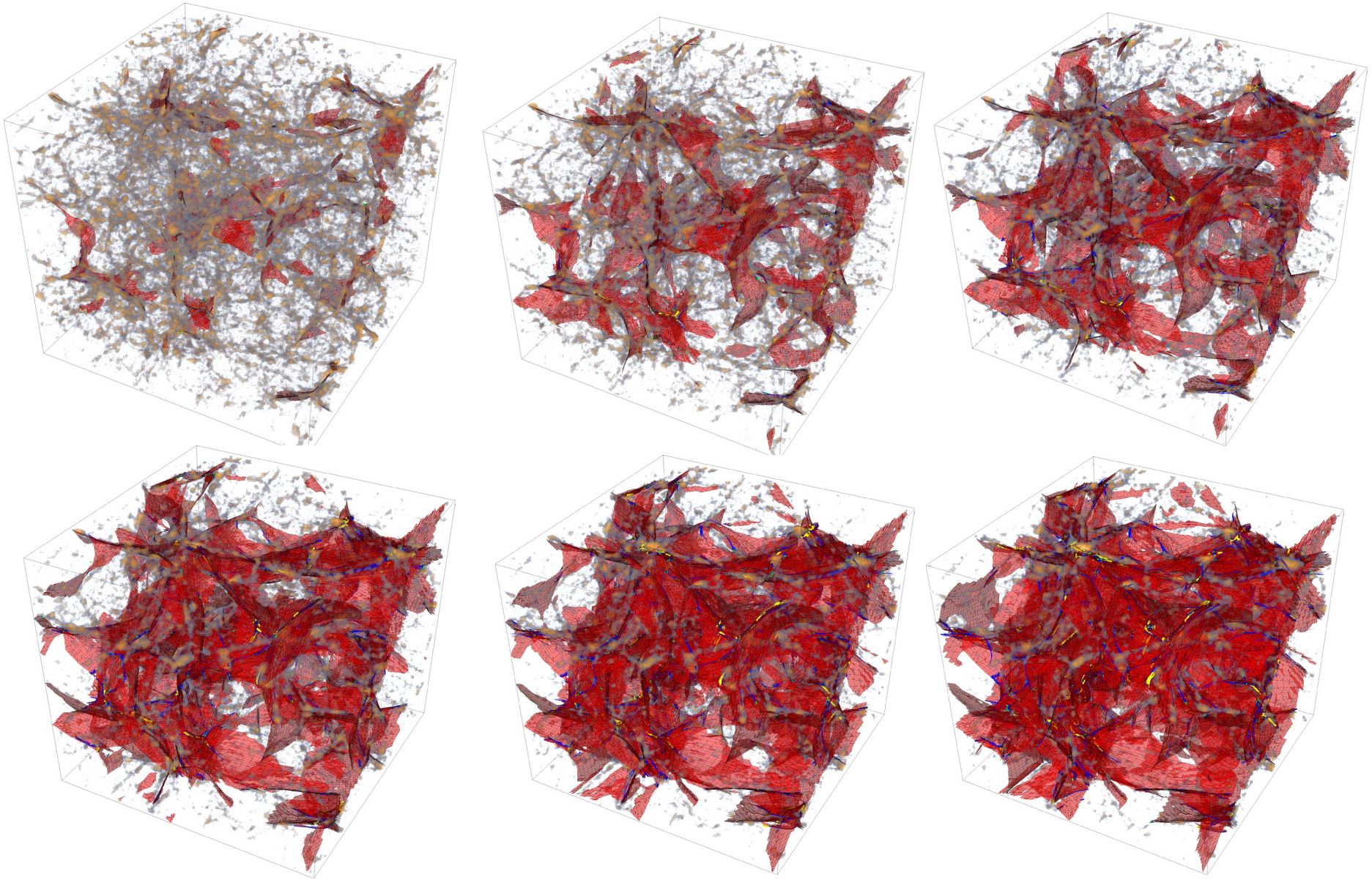




Figure 8.11: Projection: The projection of Shapley. Only the centres of the triangular facets and lines are shown here. On the left the SGY-SGZ collapsed view using Paraview and on the right the SGY-SGZ projection. Numbers indicate the cluster regions.

# Cosmic Web:
# Hierarchical Buildup of Weblike Network



# Summary

- *Analytical Model*

*Cosmic Web*

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*Caustic Skeleton*

- Shell-crossing condition enables us to derive caustic conditions in 3D
- Caustic skeleton of cosmic web depends on eigenvalue and eigenvector fields
- Filaments and walls do not require multiple shell-crossings
- We extend constrained Gaussian random field theory to non-linear constraints
- We generate constrained initial conditions tied to the dynamics of structure formation
- Using these techniques we can dress the caustic skeleton and systematically study the cosmic web
- Combining with ML, we want to study observational data